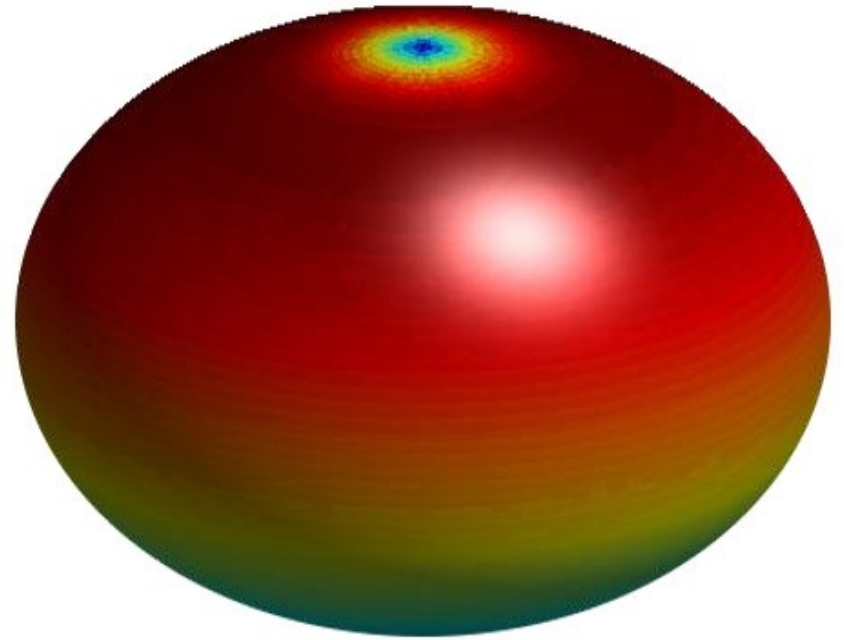


SKYRMIONS



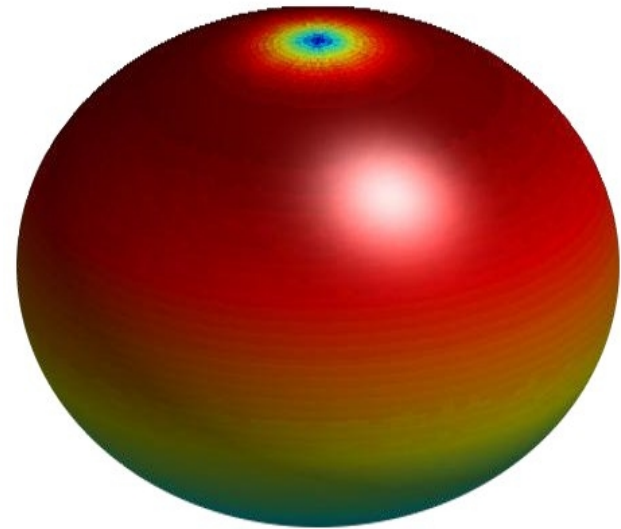
Gergely Zaránd
Márton Kanász-Nagy
Balázs Dóra

Skyrmion?

Topological Q-state

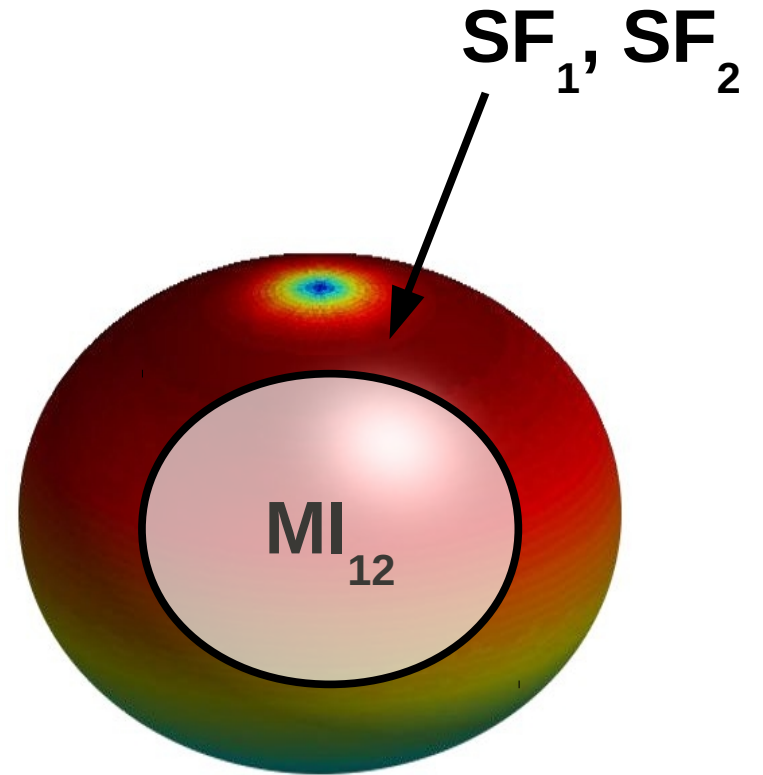
Cold atoms in a trap

2 component Bose superfluid



Skyrmion?

Topological Q-state
Cold atoms in a trap
2 component Bose superfluid



Skyrmion?

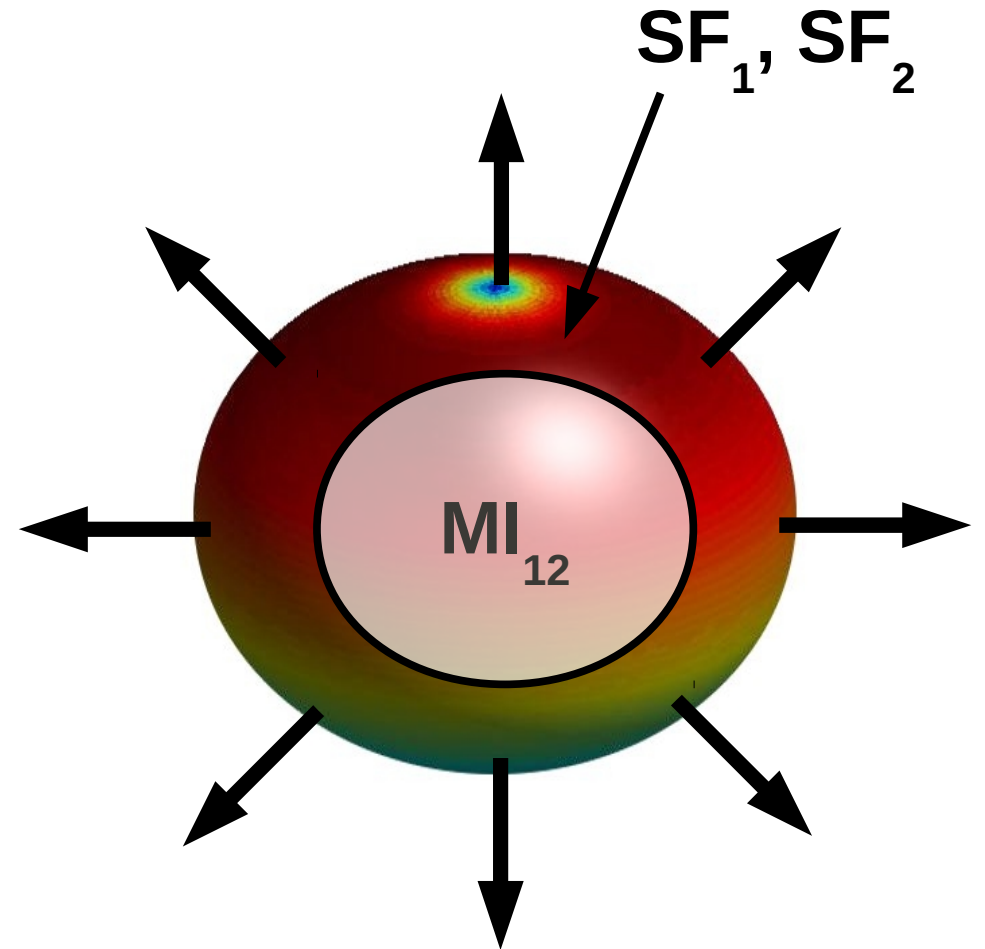
Topological Q-state
Cold atoms in a trap
2 component Bose superfluid

'Dirac-monopole on a sphere'

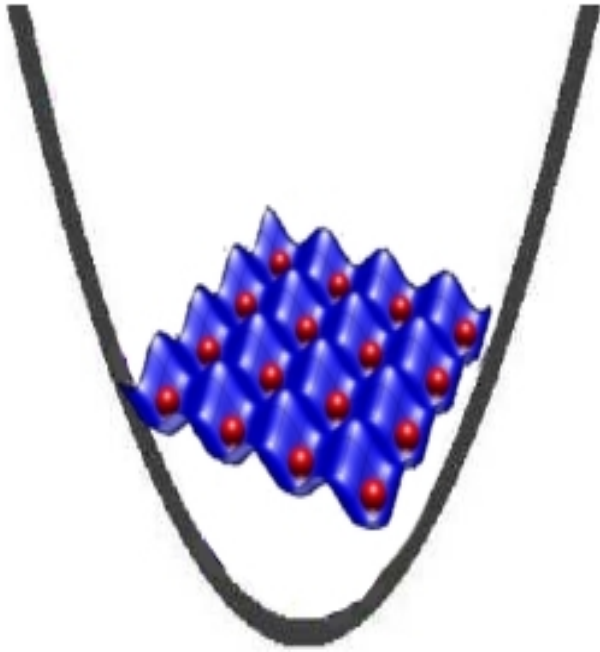
SF order parameters
 $(\psi_1, \psi_2) \longleftrightarrow$ spin

Should be

self-sustained
long lifetime
easy to manipulate



THE PHYSICAL SYSTEM



Two component bosons

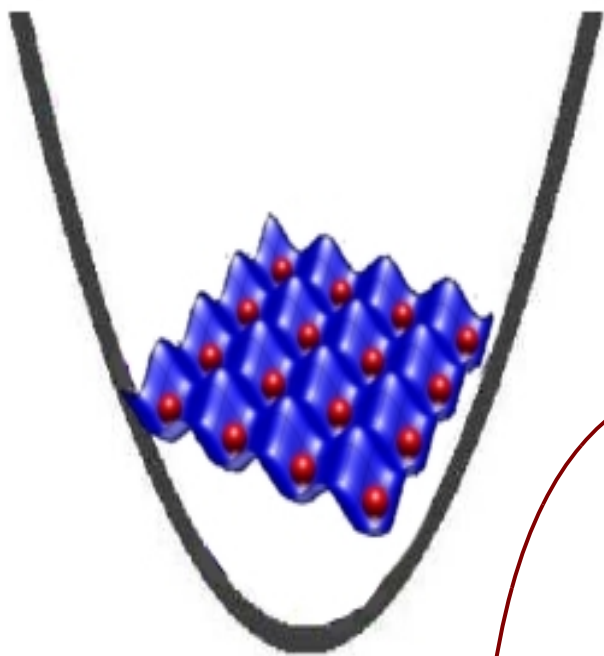
Repulsive interactions

Trapping potential

$$-\mu_{\alpha}(\mathbf{r}) = -\mu_{\alpha}^{(0)} + U_{\text{trap}}(\mathbf{r})$$

Deep optical lattices

THE PHYSICAL SYSTEM



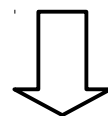
Two component bosons

Repulsive interactions

Trapping potential

$$-\mu_\alpha(\mathbf{r}) = -\mu_\alpha^{(0)} + U_{\text{trap}}(\mathbf{r})$$

Deep optical lattices



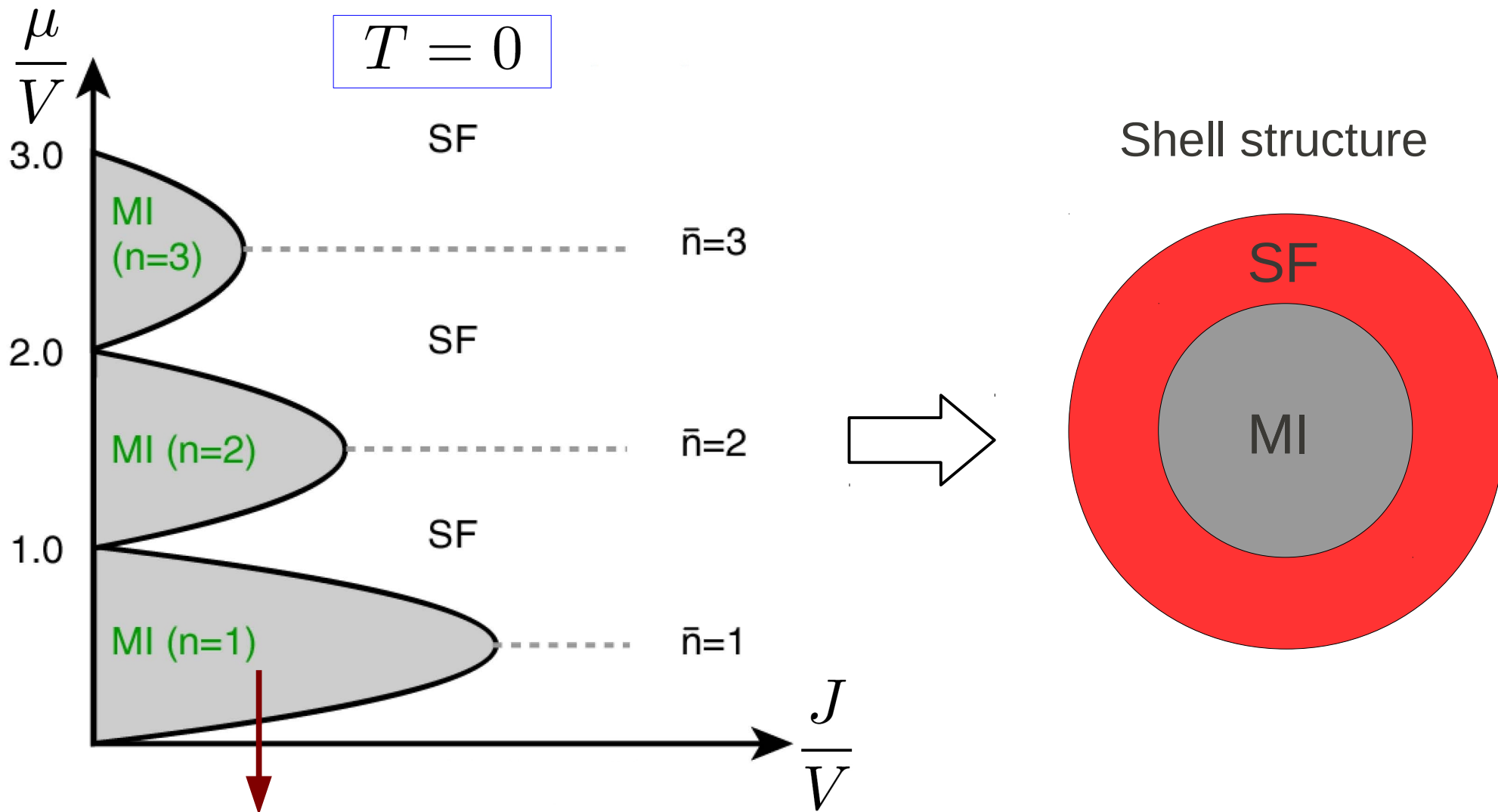
Two-component Bose-Hubbard model (one band)

$$H_0 = \sum_{\alpha i} -\mu_{\alpha i} a_{\alpha i}^\dagger a_{\alpha i} + \frac{1}{2} \sum_{\alpha \beta i} V_{\alpha \beta} a_{\alpha i}^\dagger a_{\beta i}^\dagger a_{\beta i} a_{\alpha i},$$

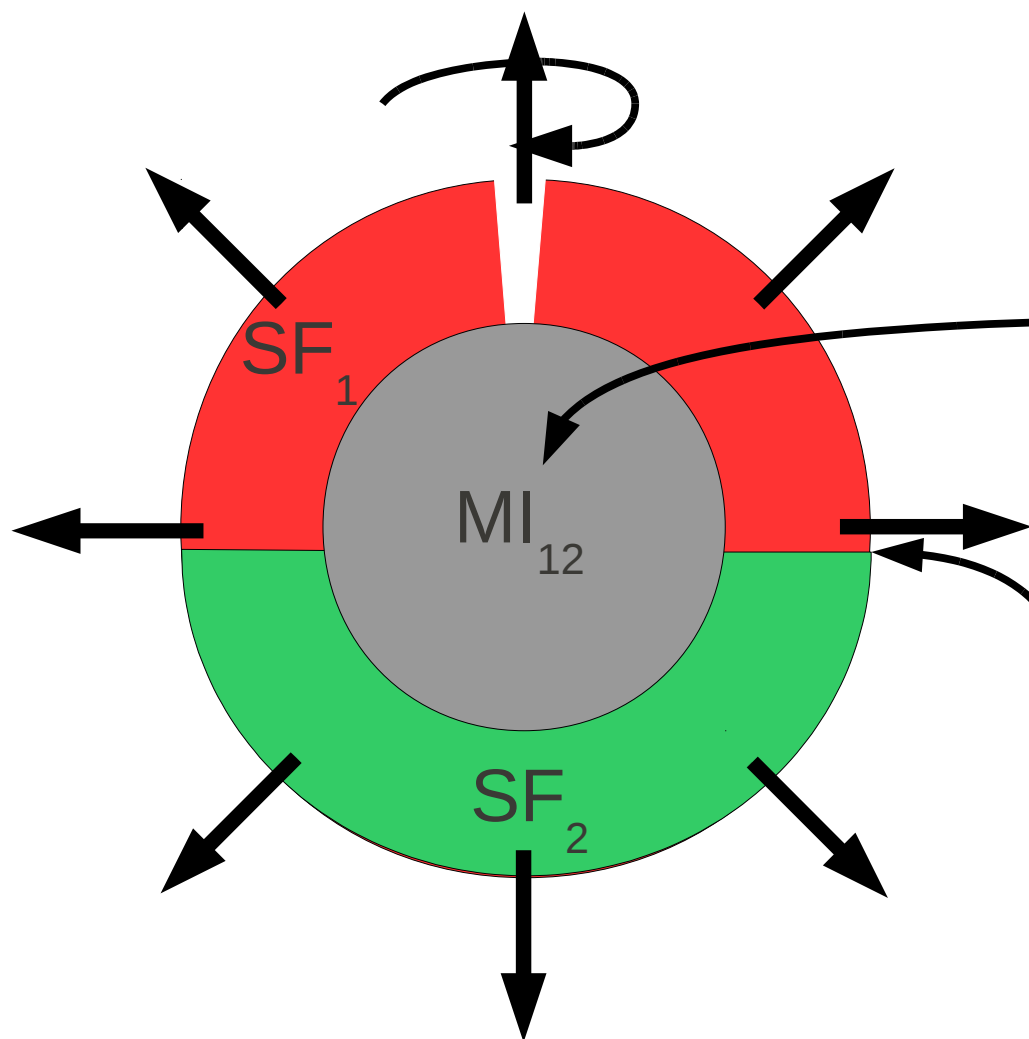
$$H_1 = - \sum_{\langle i, j \rangle \alpha} J_{ij\alpha} a_{\alpha i}^\dagger a_{\alpha j}^\dagger.$$

$$V_{11} = V_{22} < V_{12}$$

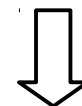
SF-MI TRANSITION FOR ONE COMPONENT



TWO COMPONENTS

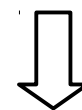


$$\frac{J^2}{V_\alpha} \ll T \ll V_\alpha$$



Homogenous mixture
in the MI phase

$$V_{11} = V_{22} \lesssim V_{12}$$

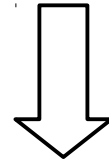


Phase separation
in the SF phase

SF order parameters: $(\psi_1, \psi_2) \iff \text{spin} \implies$ '2D Dirac monopole'

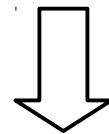
METHODS

$$H = H_0 + H_1, \quad H_1 \text{ is perturbation.}$$



Hubbard-Stratonovich,
SF order parameters: (ψ_1, ψ_2)

$$S_{\text{eff}} = \int d^3\mathbf{r} d\tau \sum_{\alpha} \left(\frac{|\nabla\psi_{\alpha}|^2}{2} + M_{\alpha}|\psi_{\alpha}|^2 + \sum_{\beta} \frac{U_{\alpha\beta}}{2} |\psi_{\alpha}|^2 |\psi_{\beta}|^2 \right)$$



Saddle point approximation

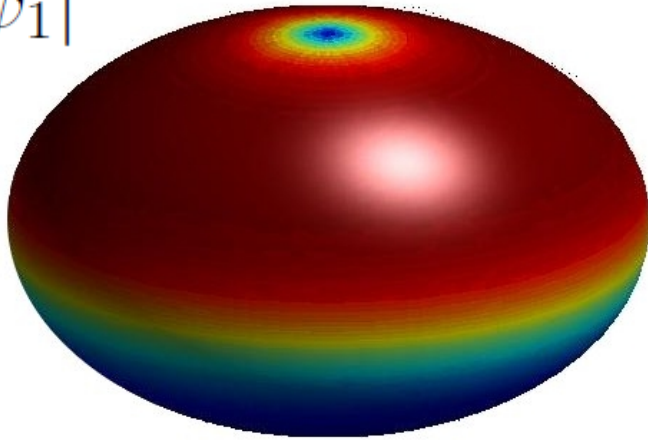
Minimization of $S_{\text{eff}} \Rightarrow \psi_{\alpha}^{\text{SP}}$

Method: imaginary time Gross-Pitaevskii equation:

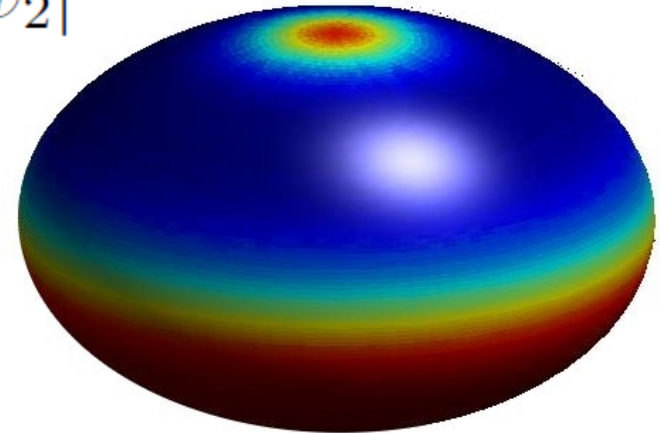
$$-\partial_{\tau}\psi_{\alpha} = \frac{\delta S_{\text{eff}}}{\delta\psi_{\alpha}}$$

PRELIMINARY RESULTS

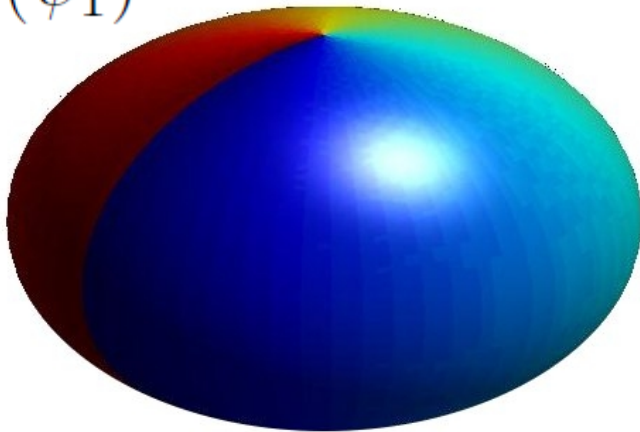
$|\psi_1|$



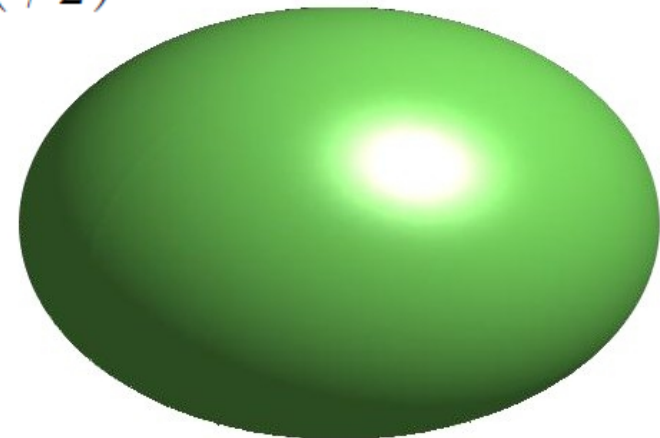
$|\psi_2|$



$\arg(\psi_1)$

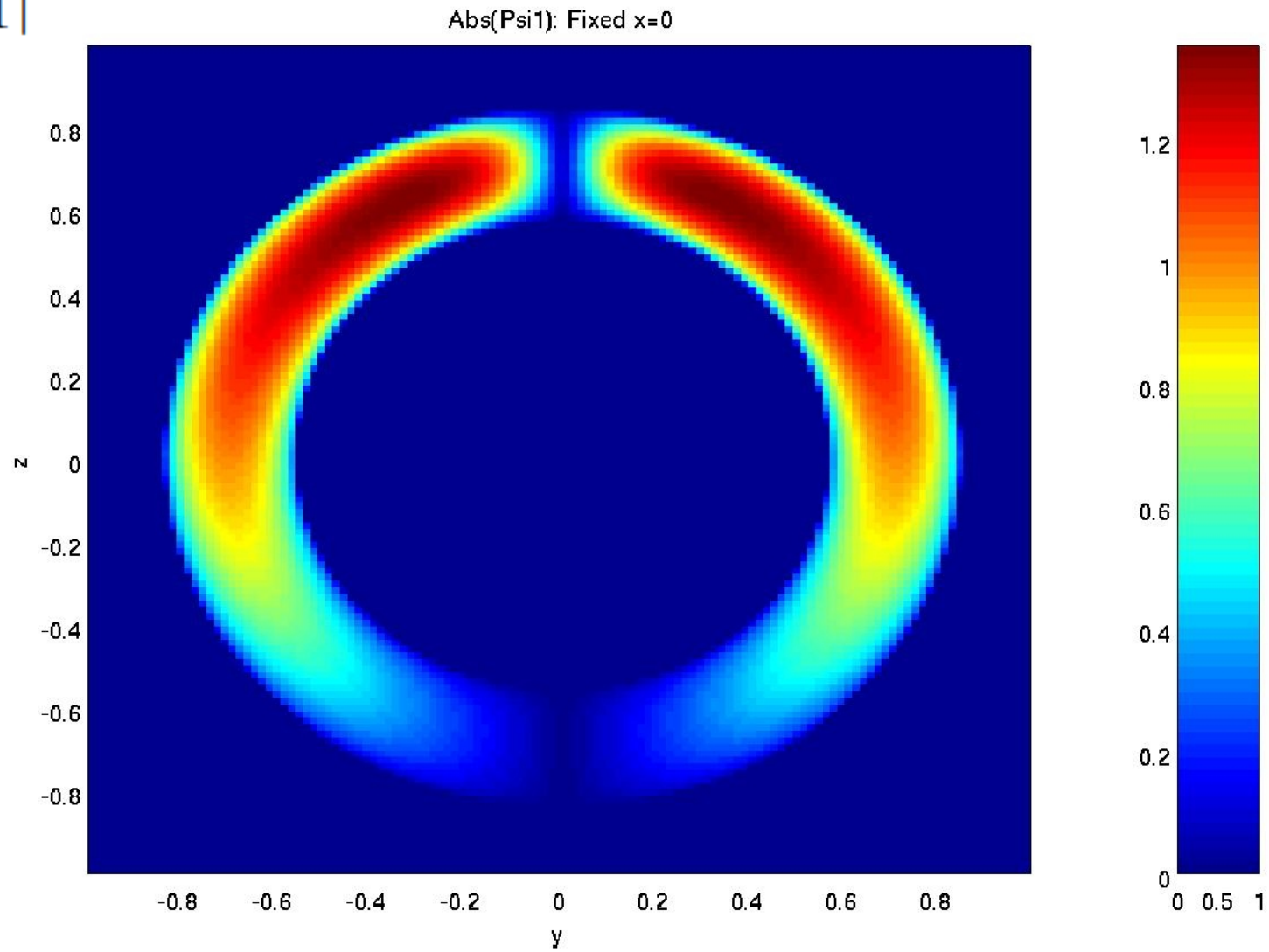


$\arg(\psi_2)$



PRELIMINARY RESULTS

$|\psi_1|$



OPEN PROBLEMS

Fix particle numbers ?

Role of the MI ?

Decay time ?

Decay processes ?

Creation of the skyrmion ?