Entanglement in the presence of defects

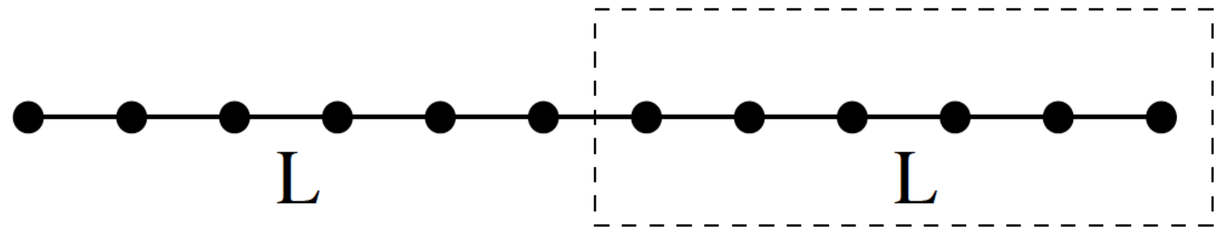
Viktor Eisler
University of Vienna

with
Ingo Peschel
FU Berlin

Entanglement in quantum ground states

- Amount of entanglement is crucial for classical simulability (DMRG, MPS, etc.)
- Quantum ground states are typically entangled: wave function cannot be written as a product
- However, they are typically not too much entangled: one has the “area law”
- Measured by von Neumann / Rényi entropies

Entanglement in homogeneous chains



- Main object: reduced density matrix

$$S = -\mathrm{tr}\,(\rho \ln \rho) \qquad\qquad S_n = \frac{1}{1-n} \ln \mathrm{tr}(\rho^n)$$

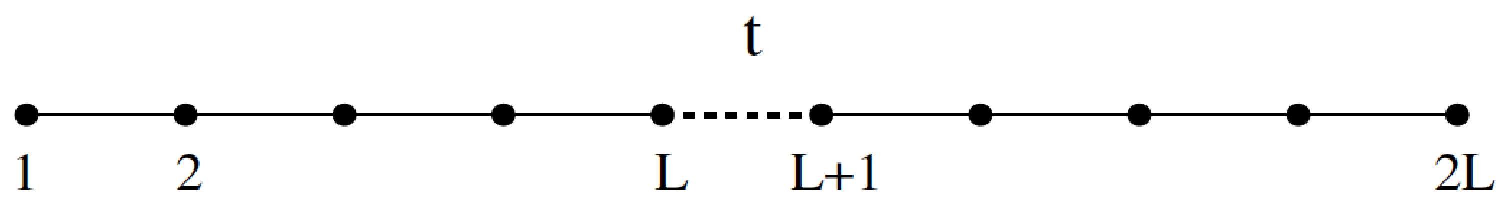
- Scaling with subsystem size:
  - c/6 ln L for critical (gapless) systems
  - c/6 ln ξ for gapped systems
- Fully understood by CFT methods
- Central charge appears in prefactor!

Free fermion/boson systems

- RDM can be written in a simple form: $\rho = \frac{1}{Z}\, \mathrm{e}^{-\mathcal{H}}$
- $\mathcal{H}$ is again a free-particle Hamiltonian
- Can be obtained through through 2-point correlations (reduced correlation matrix)
- Eigenvalues contain all the information about entanglement
- Simple formulas for entropies, e.g von Neumann:

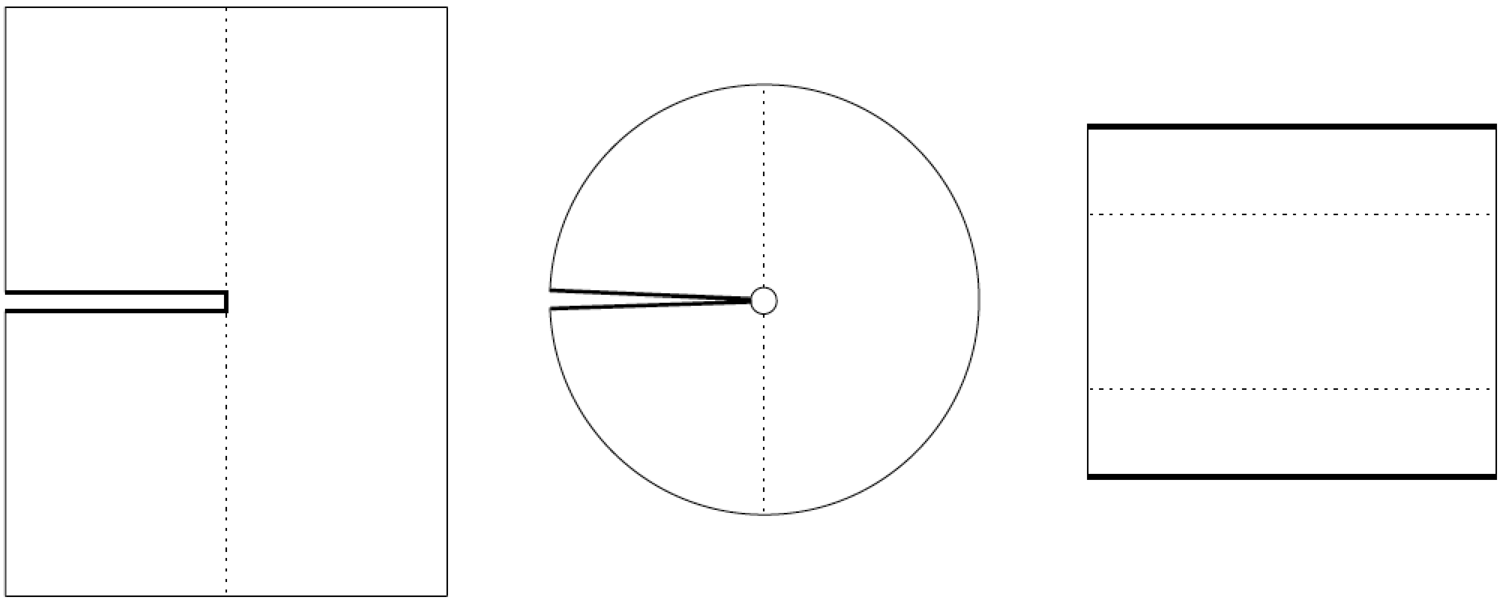
$$S = \pm \sum_k \ln(1 \pm \mathrm{e}^{-2\omega_k}) + \sum_k \frac{2\omega_k}{\mathrm{e}^{2\omega_k} \pm 1}$$

Chains with defects



- Defect breaks conformal invariance locally
- Marginal perturbation
- How does the spectrum & entropies change?
- Numerics shows: $S \sim c_{eff}/6 \ln L$
- Models considered:
  - Transverse Ising chain
  - XX chain
  - Coupled oscillators

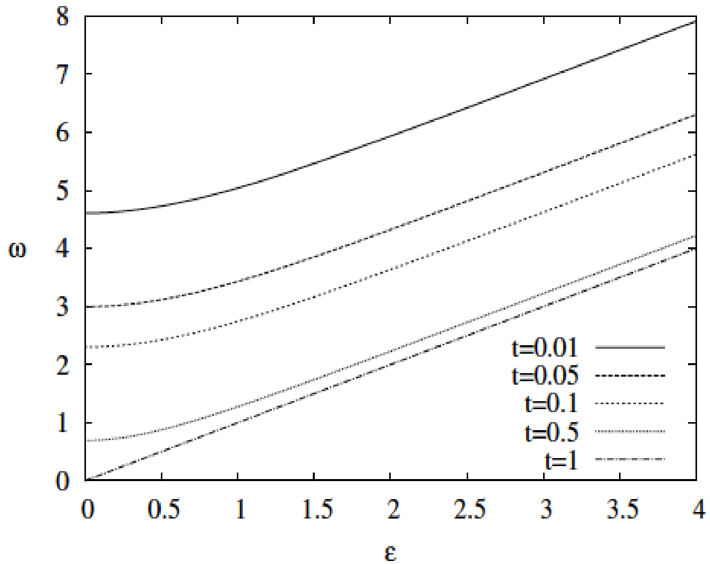
Representation of RDM



- Simpler geometry through conformal mapping
- Result: finite strip geometry
- To calculate: transfer matrix of strip with defect lines

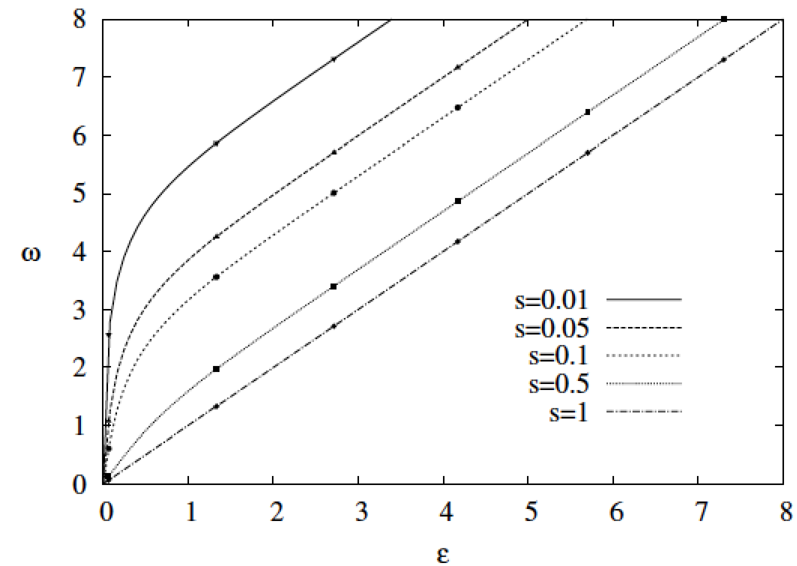
Dispersions

Fermions



$$\operatorname{ch} \omega_k = \frac{1}{s} \operatorname{ch} \varepsilon_k$$

Bosons



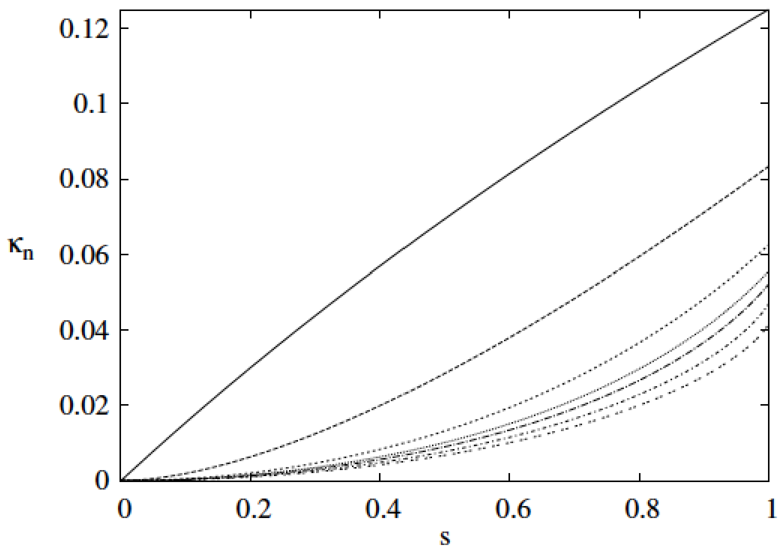
$$\operatorname{sh} \omega_k = \frac{1}{s} \operatorname{sh} \varepsilon_k$$

Parameter s corresponds to the transmission amplitude of the defect!

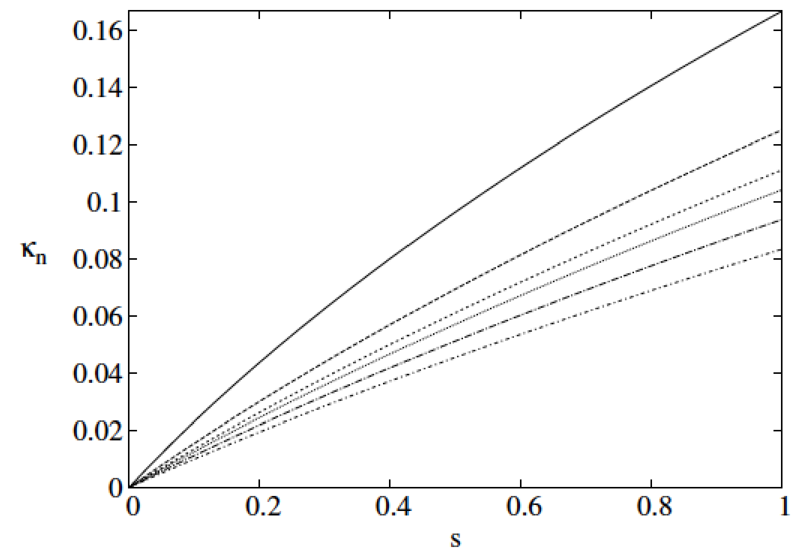
Entropies

$$S_n = \kappa_n \ln L$$

Fermions



Bosons



All the prefactors are available analytically!

$$\kappa_F(s) = -\frac{1}{2\pi^2}\{[(1+s)\ln(1+s) + (1-s)\ln(1-s)]\ln s + [(1+s)\mathrm{Li}_2(-s) + (1-s)\mathrm{Li}_2(s)]\}$$

$$\kappa(s) = \tfrac{1}{4}s - \kappa_F(s)$$

Time evolution after local quench

- What happens if we connect the chains through a defect? (Ising, XX)
- In *homogeneous* case CFT result shows:

  $S \sim c/3 \ln t$

- Central charge appears also in time evolution!
- Numerical results show for quench through defect:

  $S \sim \hat{c}_{eff}/3 \ln t$

- Is there any connection between static and dynamic effective central charges?

Conformal defect

$$H' = \frac{1}{2} \sum_{m,n=1}^{2L} H'_{m,n} c_m^\dagger c_n$$

$$H'_{m,m+1} = H'_{m+1,m} = \begin{cases} -1 & m \neq L \\ -\lambda & m = L \end{cases}$$

$$H'_{L,L} = -H'_{L+1,L+1} = \sqrt{1-\lambda^2}$$

Solved by a simple rescaling of the homogeneous eigenvectors:

$$\phi'_k(m) = \begin{cases} \alpha_k \phi_k(m) & 1 \leq m \leq L \\ \beta_k \phi_k(m) & L < m \leq 2L \end{cases} , \quad \Omega'_k = \Omega_k$$

Calculate time dependent reduced correlation matrix $\langle c_m^\dagger(t) c_n(t) \rangle$

$$\bar{\mathbf{C}}'(t) = \mathrm{e}^{i\bar{\mathbf{H}}' t} \bar{\mathbf{C}}(0) \mathrm{e}^{-i\bar{\mathbf{H}}' t}$$

$$\zeta'_l(t) = \frac{1}{\mathrm{e}^{2\omega_l(t)} + 1}$$

$$S(t) = \sum_l \ln(1 + \mathrm{e}^{-2\omega_l(t)}) + \sum_l \frac{2\omega_l(t)}{\mathrm{e}^{2\omega_l(t)} + 1}$$

$$= \sum_l H(\zeta'_l(t))$$

Start with equal fillings

- Initial correlation matrix:

$$\bar{\mathbf{C}}(0) = \begin{pmatrix} \mathbf{C}^0 & 0 \\ 0 & \mathbf{C}^0 \end{pmatrix}$$

- After time evolution one has the relation

$$(2\mathbf{C}'(t) - 1)^2_{mn} = \lambda^2 (2\mathbf{C}(t) - 1)^2_{mn} + (1 - \lambda^2)\delta_{mn}$$

  which can be rewritten using s = λ

$$\mathrm{ch}\,\omega_l(t) = \frac{1}{s}\mathrm{ch}\,\varepsilon_l(t)$$

  and leads to

$$\hat{c}_{\mathrm{eff}} = c_{\mathrm{eff}}$$

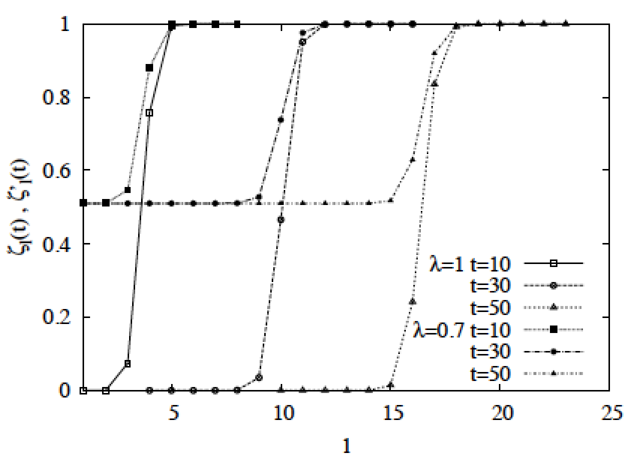
Biased case

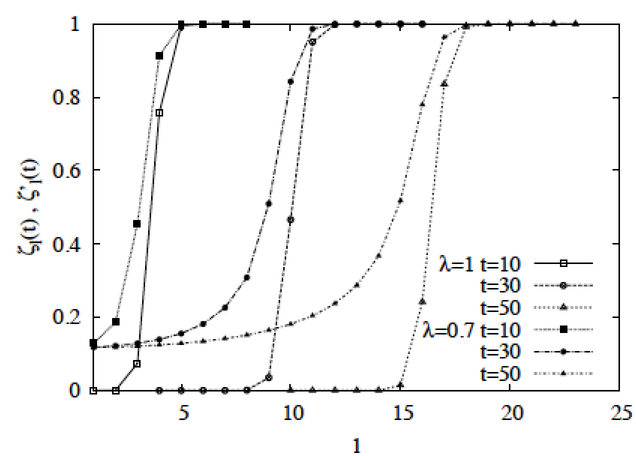
- Initial correlation matrix: $\bar{\mathbf{C}}(0) = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & 0 \end{pmatrix}$

- After time evolution:

$$C'_{mn}(t) = \lambda^2 C_{mn}(t) + (1 - \lambda^2)\delta_{mn}$$

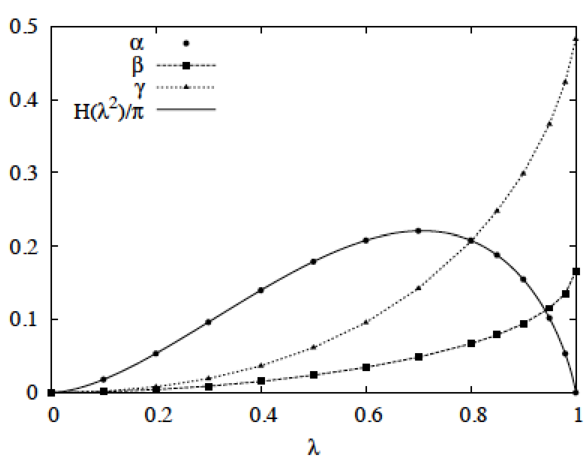
which can be rewritten as $\zeta'_l(t) = \lambda^2 \zeta_l(t) + 1 - \lambda^2$

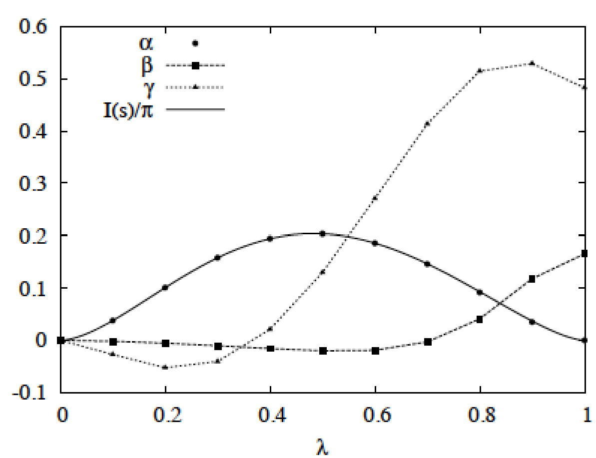




Quasi-classical picture

- Incoming particles partially transmitted / reflected
- Steady flow of particles and backscattering
- Entanglement is created steadily at the defect
- Entropy growth will be linear: $S(t) = \alpha t + \beta \ln t + \gamma$
- Ansatz for slope: $\alpha = \int_0^\pi \frac{\mathrm{d}q}{2\pi} v_q \, H(T_q)$
- Agrees perfectly with numerics:





Conclusions

- Statical defect problem solved exactly, entropy growth is logarithmic in block size
- Quench can be solved exactly for conformal defect, entropy grows linearly in time (unbiased)
- Biased case leads to a linear entropy growth!
- Entropy is generated locally but steadily
- Quasi-classical description á la

  Calabrese & Cardy / Rieger & Iglói