The Kondo effect:

Magnetic impurities in metals

Anderson:

$$H = \epsilon_d \sum_\sigma d^+_\sigma d_\sigma + U n_\uparrow n_\downarrow + V \sum_{k,\sigma} (d^+_\sigma c^+_{\sigma k} + \text{h.c.})$$
$$+ \sum_{k,\sigma} \epsilon_k \, c^+_{k\sigma} c_{k\sigma}$$

$U = 0$: resonant level model

$$\mathcal{G}^{(0)}(i\omega_n) = \frac{1}{i\omega_n - \epsilon_d}$$

$$\mathcal{G}^{(0)}_k = \frac{1}{i\omega_n - \epsilon_k} \qquad \xrightarrow{k}$$

$V \Rightarrow$ or

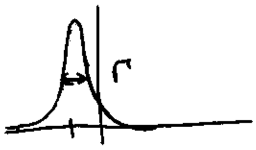
self-energy: $\quad \Rightarrow \quad \Sigma(i\omega) = \sum_k V^2 \frac{1}{i\omega - \epsilon_k} =$

$$= V^2 \int d\xi \, \rho(\xi) \frac{1}{i\omega - \xi} = -i\pi \rho(0) V^2 \operatorname{sgn}\omega$$

$$\Rightarrow \quad \boxed{\mathcal{G}(i\omega_n) = \frac{1}{i\omega_n - \epsilon_d + i\frac{\Gamma}{2}\operatorname{sgn}\omega_n}} \qquad \Gamma = 2\pi\rho V^2$$

↑ rate of escape

$$G^R(\omega) = \frac{1}{\omega - \epsilon_d + i\frac{\Gamma}{2}} \quad \Rightarrow \quad \rho(\omega) = \frac{\Gamma}{2\pi} \cdot \frac{1}{(\omega-\epsilon_d)^2 + \Gamma^2/4}$$



Γ

ϵ_d

$$\langle n_\uparrow \rangle = \langle n_\downarrow \rangle = \int \frac{d\omega}{2\pi} \frac{\Gamma}{(\omega-\epsilon_d)^2 + \Gamma^2/4} \cdot f(\omega)$$

↑ Fermi function

$$T = 0: \quad \langle n_\uparrow \rangle = \frac{1}{\pi}\left(\arctan\left(\frac{-\epsilon_d}{\Gamma/2}\right) + \frac{\pi}{2}\right)$$

Anderson: MF solution

$$U n_\uparrow n_\downarrow \longrightarrow U n_\uparrow \langle n_\downarrow \rangle + U n_\downarrow \langle n_\uparrow \rangle .$$

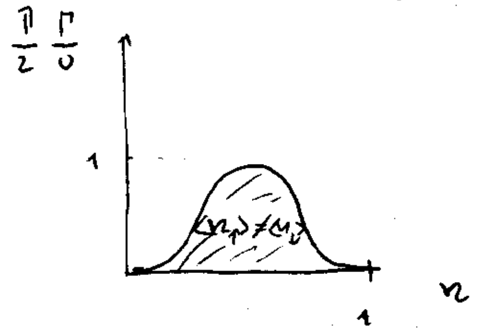
$$\epsilon_d \rightarrow \epsilon_{\uparrow,\downarrow} = \epsilon_d + U \langle n_{\downarrow,\uparrow} \rangle$$

$$\langle n_\uparrow \rangle = \frac{1}{\pi} \left\{ \frac{\pi}{2} - \arctan \frac{2\epsilon_\uparrow}{\Gamma} \right\} = \frac{1}{2} - \frac{1}{\pi} \operatorname{atan} \frac{2}{\Gamma}(\epsilon + U \langle n_\downarrow \rangle))$$

stability analysis $\quad n_{\uparrow,\downarrow} = \delta n_{\uparrow,\downarrow} + n/2$

$\langle n_\uparrow \rangle = \langle n_\downarrow \rangle$ solution unstable if

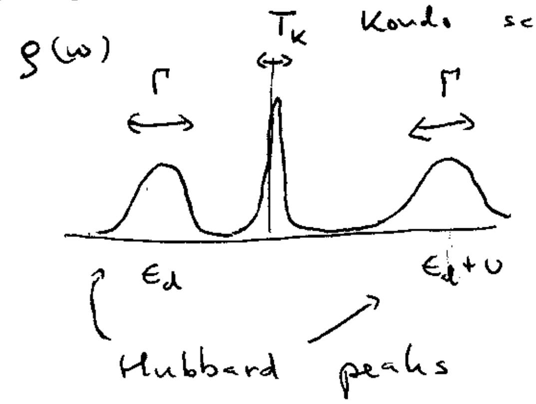
$$\frac{U\Gamma}{2\pi} \cdot \frac{1}{\frac{\Gamma^2}{4} + \tilde{\epsilon}^2} > 1 \iff \boxed{\frac{\pi \cdot \Gamma}{2U} < \sin^2\left(\frac{\pi n}{2}\right)}$$



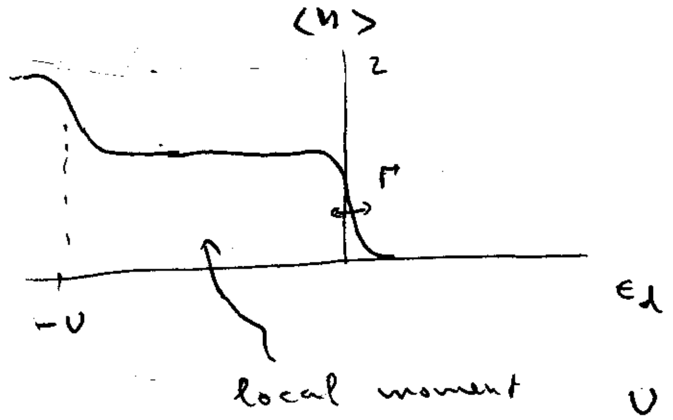
$\rightarrow$ local moment formation
quantum fluctuations important ?

Reality

T_K Kondo scale $\iff$ quantum fluctuations of local moment



$\Leftarrow$ Numerical RG (NRG)
perturbation theory ...

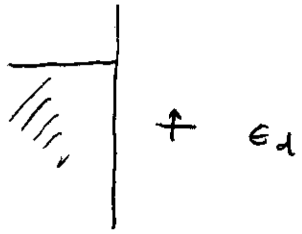


$\Leftarrow$ Bethe Ansatz
NRG

$U \gg \Gamma$

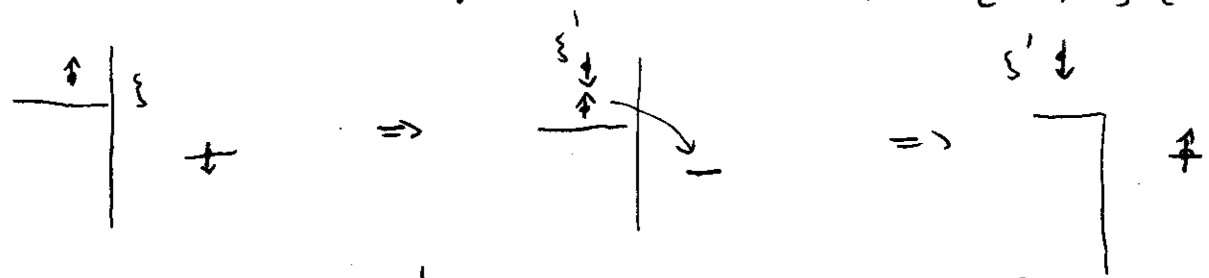
$-U < \epsilon_d < 0$

Kondo model:



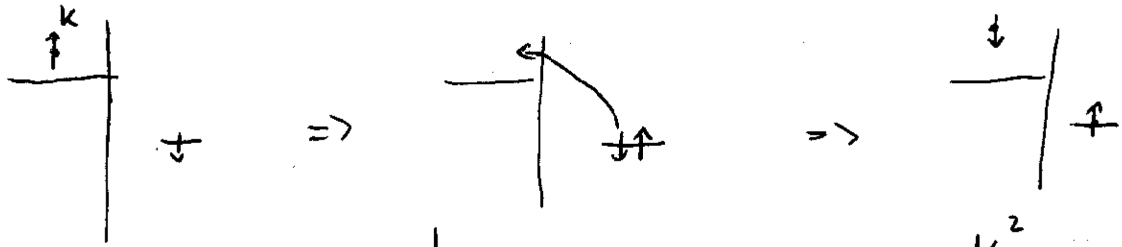
2nd order Pert. theory in subspace $\{|k_1\sigma_1,\dots\rangle\} \otimes \{|\uparrow\rangle, |\downarrow\rangle\}$ ← Fermi sea

(a)



$$\sim \quad V d^+_\uparrow c_{k\uparrow} \frac{1}{\xi_k + \epsilon_d - \xi_{k'}} V c^+_{k'\downarrow} d_\downarrow = -\frac{V^2}{\epsilon_d} c^+_{k'\downarrow} c_{k\uparrow} d^+_\uparrow d_\downarrow$$

(b)



$$V c^+_{k'\downarrow} d_\downarrow \frac{1}{\xi_k + \epsilon_d - (2\epsilon_d + U)} V d^+_\uparrow c_{k\uparrow} \cong \frac{V^2}{\epsilon_d + U} c^+_{k'\downarrow} c_{k\uparrow} \underbrace{d^+_\uparrow d_\downarrow}_{S^+}$$

$$\Rightarrow \boxed{H_{eff} = \frac{1}{2} J \sum_{k,k'} \vec{S} \cdot (c^+_k \vec{\sigma} c_{k'})} \quad \text{Kondo model}$$

$$H_K \qquad \frac{J}{2} = V^2 \left(\frac{1}{\epsilon_d + U} - \frac{1}{\epsilon_d} \right)$$

dimensionless coupling: $j \equiv J \rho_0 = \frac{\Gamma}{\pi} \left(\frac{1}{\epsilon_d + U} - \frac{1}{\epsilon_d} \right) > 0$!

AF coupling!

H_{eff} contains log singularity!

T = 0 temperature

compute on-shell T matrix:

$$|i\rangle \equiv c^+_{k\alpha}|s\rangle \qquad 0 < \xi_k = \xi_{k'} \quad (E_i = E_f)$$
$$k \neq k'$$
$$|f\rangle \equiv c^+_{k'\beta}|s'\rangle \leftarrow |FS\rangle \otimes |s'\rangle$$

$$\langle f|\tau|i\rangle =$$

$$= \langle f|H_K|i\rangle + \langle f|H_K \frac{1}{E_i - H_0} H_K|i\rangle$$

① $$\langle s'| \frac{J}{2} \sum_{\substack{k_1 k_1' \\ \sigma_1 \sigma_1' \\ i}} c_{k'\beta} c^+_{k_1\sigma_1} c_{k_1'\sigma_1'} c^+_{k\alpha} \sigma^i_{\sigma_1\sigma_1'} S^i |s\rangle$$
$$= \frac{J}{2} \vec{\sigma}_{\beta\alpha} \cdot \vec{S}_{s's}$$

② $$\left(\frac{J}{2}\right)^2 \sum_{ij} \sum_{k_1 \dots} \sum_{k_2 \dots} \langle s'| c_{k'\beta} c^+_{k_1\sigma_1} c_{k_1'\sigma_1'} S^i \frac{1}{\xi_k - H_0} c^+_{k_2\sigma_2} c_{k_2'\sigma_2'} S^j c^+_{k\alpha} |s\rangle \sigma^i_{\dots} \sigma^j_{\dots}$$

possibility a:

$$\Rightarrow \left(\frac{J}{2}\right)^2 \sum_{\substack{k_2 \\ \xi_{k_2} > 0}} \frac{1}{\xi_k - \xi_2} (S^i S^j)_{s's} \underbrace{(\sigma^i \sigma^j)_{\beta\alpha}}_{\delta^{ij} + i\epsilon^{ijk}\sigma^k}$$

$$\downarrow$$
$$(S(S+1) - \bar{S}\cdot\bar{\sigma})_{s's,\beta\alpha}$$

possibility b:

$$\left(\frac{J}{2}\right)^2 (-1) \sum_{\substack{k_1 \\ \xi_1 < 0}} \underbrace{\frac{1}{\xi_k - (-\xi_{k_1} + \xi_k + \underset{\xi_k}{\xi_{k'}})}}_{\frac{-1}{\xi_k - \xi_1}} \underbrace{(S^i S^j)_{s's} (\sigma^j \sigma^i)_{\beta\alpha}}_{S(S+1) + \bar{S}\cdot\bar{\sigma}}$$

$$\Rightarrow \left(\frac{J}{2}\right)^2 \sum_{\xi'} \frac{1}{\xi_k - \xi'} \overbrace{S(S+1)}^{\approx \emptyset}$$

$$+ \left(\frac{J}{2}\right)^2 \underbrace{\sum_{\xi'} \frac{1}{\xi' - \xi_k} \operatorname{sgn} \xi'}_{} \quad \vec{\sigma} \cdot \vec{S}$$

$$\hat{=} \rho(0) \int_{-D}^{D} d\xi' \frac{1}{\xi' - \xi} \operatorname{sgn} \xi' \;\hat{=}\; 2\rho(0) \ln \frac{D}{|\xi|}$$

$$\boxed{\langle f | T | i \rangle \hat{=} \frac{1}{2}\left(J + J^2 \rho(0) \ln \frac{D}{|\xi|}\right) \vec{S}_{s's} \cdot \vec{\sigma}_{\beta\alpha}}$$

this is related to scattering cross-section

$\Rightarrow$ must be R.G. inv.

$D \to D' < D$ $\qquad J' = J + J^2 \rho_0 \ln \frac{D}{D'}$

$J \to J'$

$$x \hat{=} \ln \frac{D_0}{D} \Rightarrow \boxed{\frac{dj}{dx} = j^2 + \ldots} \qquad (*)$$

$j > 0 \Rightarrow$ relevant $\Rightarrow$ scales to ∞

solution $\qquad j(\tilde{x}) = \frac{j_0}{1 + \tilde{x} \cdot j_0} = \underset{\sim\sim\sim}{\frac{1}{\ln\left(\frac{\tilde{D}}{T_K}\right)}} \qquad \underset{\sim\sim\sim}{T_K = D_0 e^{-\frac{1}{j_0}}}$

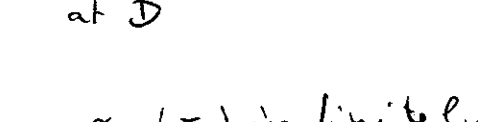
eff coupling at D

for $|\xi| \to \emptyset$ (T_K) infinitely strong scattering

$J(\xi) \to \infty \quad \Rightarrow \quad J(T \sim \xi) \sim \frac{1}{\ln \frac{T}{T_K}}$

$\Rightarrow$ resistivity $\sim \underline{\frac{1}{\ln^2 \frac{T}{T_K}}}$!

$\Rightarrow$ local singlet formation:



(Fermi liquid: Nozières)