Ultracold lattice gases with periodically modulated interactions


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Outline

1. Ultracold atoms in optical lattices
2. Periodic modulations
3. Effective Floquet Hamiltonian
4. Real-time dynamics
5. Summary
Ultracold atoms in optical lattices

Optical lattices

- standing wave from non-resonant laser effective periodic potential
  \[ V(x) = V_L \sin^2 k_L x \]
- deep optical lattices nearest-neighbor hopping \( J = J(V_L) \)
Ultracold atoms in optical lattices

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Interactions

- relevant parameter: scattering length \( a_s \)
- changing \( a_s \): by Feshbach resonances
  Chin et al. Rev. Mod. Phys. 82, 1225 (2010)
- optical lattices: local interactions \( U \sim a_s \) → Hubbard models
Hubbard models for ...

(strongly) correlated solid state systems
- presence of impurities, phonons, etc...
- microscopic parameters are not easy to tune and to determine
- fast (electronic) time scales
Ultracold atoms in optical lattices

Hubbard models for ...

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Hubbard models for ...

ultracold atoms in optical lattices

- "prefect crystal"
- excellent control of microscopic parameters
- relatively long time scales tracking systems in real time

⇒ out-of-equilibrium

- quantum quenches
- lattice shaking
- ...

Á. Rapp: Periodically modulated interactions
Optical lattices – with **shaking**
Optical lattices – with shaking

Lattice shaking $\rightarrow$ Floquet analysis $\rightarrow$ rescaled hopping

$$J \rightarrow J_{\text{eff}} = J J_0 \left( \frac{k_1}{\omega} \right)$$

Eckart et al., PRL 95, 260404 (2005), Lignier et al., PRL 99, 220403 (2007), Zenesini et al., PRL 102, 100403 (2009), ...
Application of lattice shaking:

Quantum Simulation of Frustrated Classical Magnetism in Triangular Optical Lattices


Struck et al., Science 333, 996 (2011)
Interactions – with periodic modulation in time
Interactions – with periodic modulation in time

Modulation of the $B \rightarrow a_s$ modulation of $a_s$

- excite quadrupole mode in a harmonic trap (experiment)
  Pollack et al., PRA 81, 053627 (2010)
- bosons in a double-well potential (theory)
  Gong et al., PRL 103, 133002 (2009)
Interactions – with periodic modulation in time

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Here:
- Effective theory & phase diagram for the lattice
- Real-time dynamics & experimental signature
Periodically modulated interactions

- time-dependent Bose-Hubbard Hamiltonian with $U \rightarrow U(t) \approx U_0 + U_1 \cos(\omega t)$

$$H(t) = -J \sum_{\langle ij \rangle} b_i^{\dagger} b_j + \frac{U(t)}{2} \sum_i \hat{n}_i (\hat{n}_i - 1)$$
Periodically modulated interactions

- Time-dependent Bose-Hubbard Hamiltonian with \( U \rightarrow U(t) \approx U_0 + U_1 \cos(\omega t) \)

\[
H(t) = -J \sum_{\langle ij \rangle} b_i^+ b_j + \frac{U(t)}{2} \sum_i \hat{n}_i (\hat{n}_i - 1)
\]

- Floquet analysis
  1. take \( H_0(t) = U(t) \times \ldots \) and \( H_1 = H(t) - H_0(t) \)
  2. solve time-dependent Schrödinger equation \textit{analytically}

\[
[i\hbar \partial_t - H_0(t)] |\psi_0(t)\rangle = \epsilon |\psi_0(t)\rangle
\]

- Transform to the "interaction picture" with Floquet evolution operator

\[
\mathcal{U}(t) = e^{-i \int_0^t dt' H_0(t')/\hbar}
\]

- Derive effective \textit{time-independent} Hamiltonian by time averaging

\[
H_{\text{eff}} = \frac{1}{T} \int_0^T dt \mathcal{U}^+(t) H_1 \mathcal{U}(t)
\]
Periodically modulated interactions

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- Floquet analysis $\rightarrow$ effective time-independent Hamiltonian for $J, U_0 \ll \hbar \omega$

$$H_{\text{eff}} = -J \sum_{\langle ij \rangle} b_i^+ J_0 \left( \frac{U_1}{\hbar \omega} (\hat{n}_i - \hat{n}_j) \right) b_j + \frac{U_0}{2} \sum_i \hat{n}_i (\hat{n}_i - 1)$$

Á. Rapp: Periodically modulated interactions

Budapest
Periodically modulated interactions

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- main difference w.r.t. lattice shaking: non-linearity in occupation differences

- most interesting regime:

$$U_1 \gtrsim \hbar \omega$$
$U_1/\hbar\omega = 4 \rightarrow J_0(U_1/\hbar\omega) < 0$

Gutzwiller ansatz

DMRG

MI  Mott insulator (not really affected)

SF  “usual” superfluid

\[ \frac{|\langle \hat{b} \rangle|}{\sqrt{n}} > \frac{|\langle \hat{b}^2 \rangle|}{n} \]

PSF  pair superfluid

\[ \frac{|\langle \hat{b} \rangle|}{\sqrt{n}} \leq \frac{|\langle \hat{b}^2 \rangle|}{n} \]
Effective Hamiltonian – phase diagram

\[ \frac{U_1}{\hbar \omega} \approx 2.4 \rightarrow J_0(\frac{U_1}{\hbar \omega}) \approx 0 \]

- **MI**: no defects even in \( d = 1! \)
- **holon/doublon SF**: holes and doublons do not mix!
We have analyzed so far the effective system. But what happens in the real system?
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Gutzwiller Ansatz

- wave function ansatz: \( |G(t)\rangle = \prod_j |f_m(j, t)|m\rangle \)
- time-dependent variational principle: \( \langle G(t)|i\hbar \partial_t - H(t)|G(t)\rangle \to \text{min} \)
- dynamical system:

\[
i\hbar \partial_t f_m(j, t) = \left[ U(t) \frac{m(m-1)}{2} + (V(r_j, t) - \mu_0)m \right] f_m(j, t)
- J(t)\Phi^*(j, t)\sqrt{m+1} f_{m+1}(j, t) - J(t)\Phi(j, t)\sqrt{m} f_{m-1}(j, t)
\]

- mean-field \( \Phi(j, t) = \sum_{\delta} \langle G|\hat{b}_{j+\delta}|G\rangle \)
- microscopic parameters \( U, J, \) and \( V \) can change as a function of time \( t \)
- provide initial condition \( f_m(j, t = 0) \) also from GA
Real-time dynamics

Possible experimental procedure

1. start with $B_1 = 0$ and ramp up adiabatically

2. careful with energy scales!

\[ J, U_0 \ll \hbar \omega \lesssim U_1 \ll \text{band gap} \]

- small $E_R$ (large $M$, large $\lambda_L$)
- zero crossing near wide resonance

![Feshbach resonances in $^{133}$Cs](image)

resonance data from Manfred Mark, private comm.
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Here:

- 3D cubic lattice with $N_{\text{tot}} \approx 43 \times 10^3$ $^{133}$Cs atoms
- Lattice depth $V_L/E_R = 20 \rightarrow$ hopping rate $J/E_R \approx 0.002491$
- Final magnetic field: $B(t) = B_0 + B_1 \cos(\omega t)$,

$$\omega = 2\pi \times 1 \text{ kHz}, B_0 = 17.23 \text{ G}, B_1 = 1.14 \text{ G} \rightarrow \frac{U_1}{\hbar \omega} \approx 2.4, J_0 \left( \frac{U_1}{\hbar \omega} \right) \approx 0$$
Real-time dynamics

What should we look for? Abrupt drops in *in-situ* density profile

$t=0.0 \text{ ms (} \omega = 2\pi \times 1 \text{ kHz)}$

![Graph showing n(r) and d(r) profiles over time](image)
Real-time dynamics

What should we look for? Abrupt drops in *in-situ* density profile

![Graph showing density profile over time](image)

- $t = 90.0 \text{ ms}$ ($\omega = 2\pi \times 1 \text{ kHz}$)

Legend:
- $n(r)$
- $d(r)$
Real-time dynamics

What should we look for? Abrupt drops in *in-situ* density profile

t=150.0 ms (ω = 2π × 1 kHz)

![Graph showing n(r) and d(r) over time](image)
Real-time dynamics

What should we look for? Abrupt drops in *in-situ* density profile

![Graph showing in-situ density profile](image)

$t=200.0$ ms ($\omega = 2\pi \times 1$ kHz)
Real-time dynamics

What should we look for? Abrupt drops in *in-situ* density profile

3D cloud → *in situ* images show the *columnar* atom density
Summary

- periodic modulation of interactions
  PRL 109, 203005 (2012)
  – new experimental KNOB?

- main issue: HEATING
  → need for real-time simulations

Thank you for the attention!