Crossover from adiabatic to sudden interaction quench in a Luttinger liquid

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Outline:
- Non-equilibrium dynamics in cold atoms
- Luttinger liquid in equilibrium
- Bogoliubov coefficients out of equilibrium
- Heating after the quench
- One particle density matrix, spatial-temporal "phase diagram"

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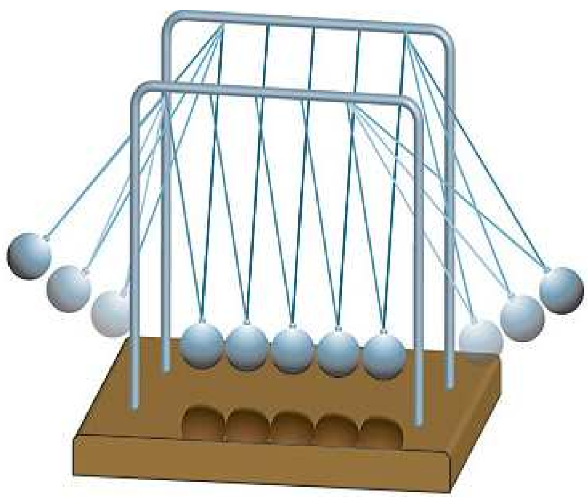
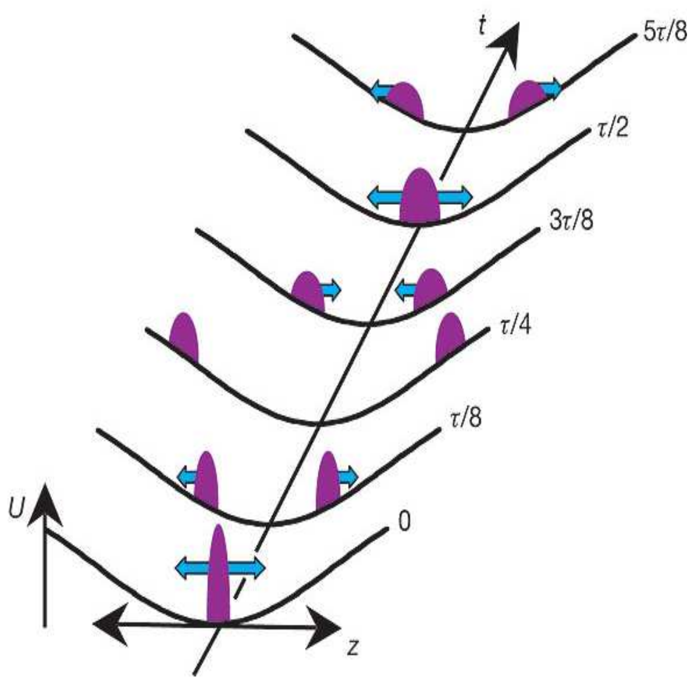
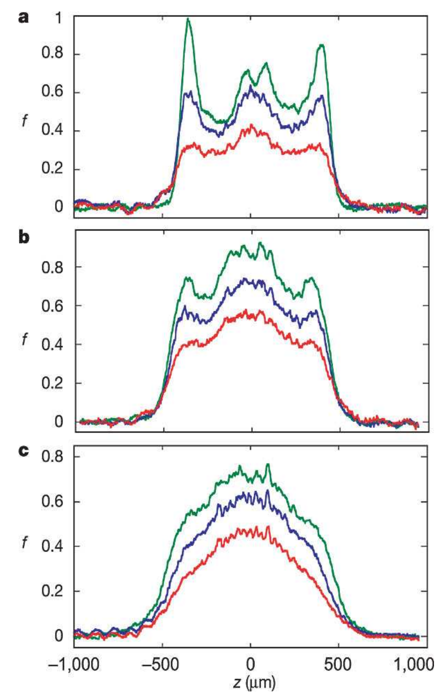
Non-equilibrium cold atoms in 1D

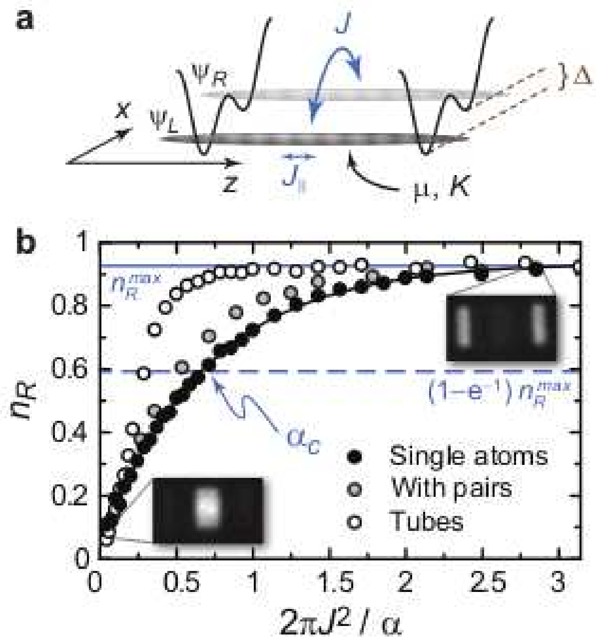
- 1D Bose gas (^{87}Rb) atoms out of equilibrium
- No noticable equilibration even after thousands of collisions
- Absence of damping, homogeneous 1D Bose gas with point-like interactions is integrable



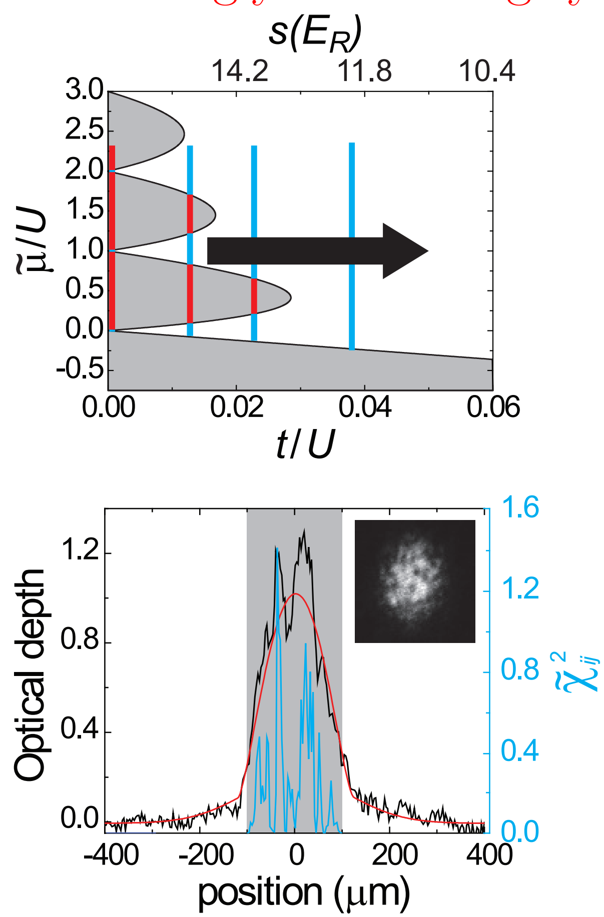




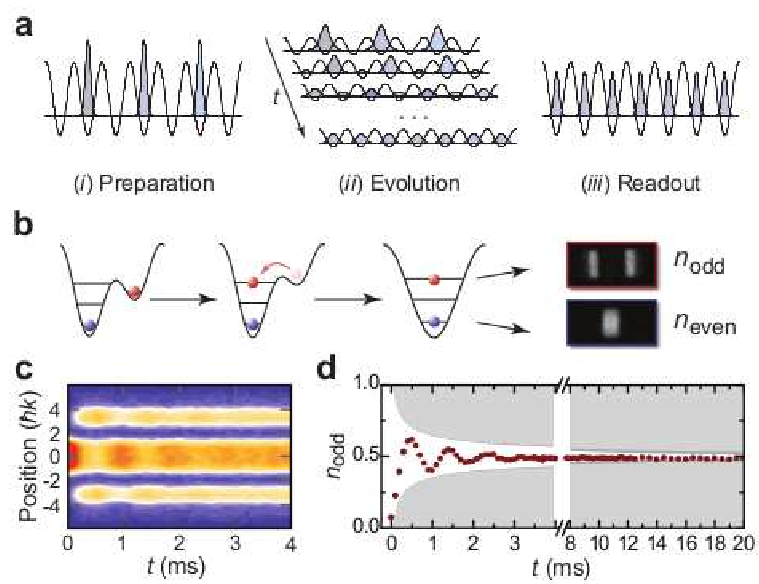
Quantum quenches in strongly interacting systems



Coupled 1D Bose liquids



Interaction quench in 3D Bose systems



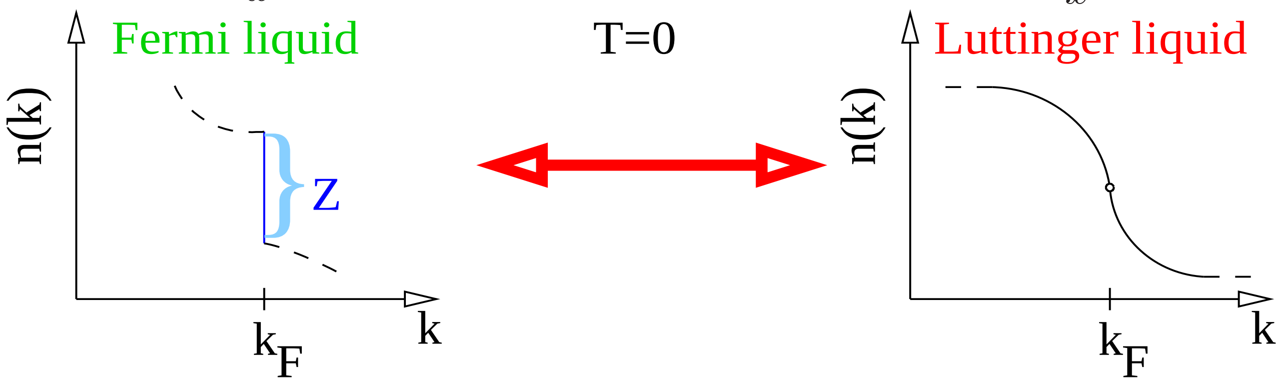
Relaxation from CDW

Luttinger liquid in equilibrium

Interacting 1D electron gas: metallic or gapped, Fermi liquid picture breaks down.

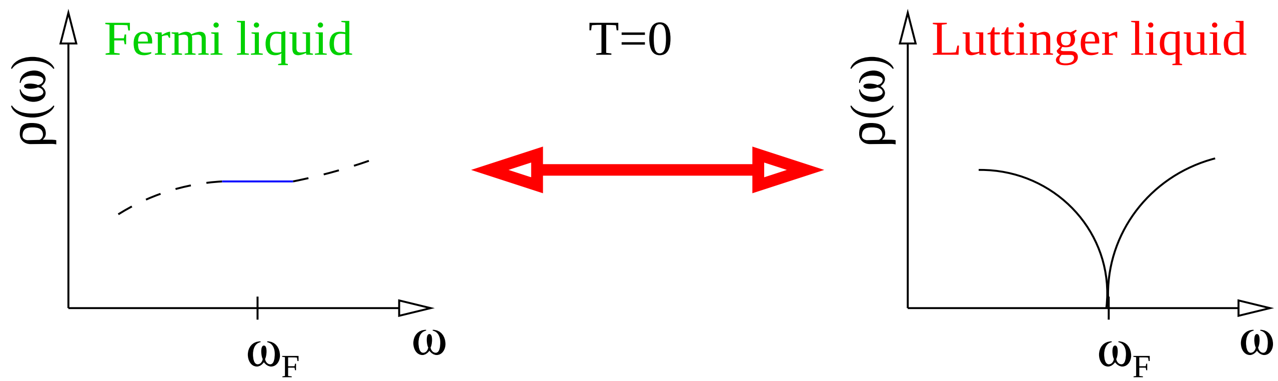
$\langle \Psi(x,t)\Psi^+(0,t)\rangle \sim \quad \frac{Z}{x} \quad \Longleftrightarrow \quad \frac{1}{x^{1+\gamma^2}}$

momentum distribution:



$\langle \Psi(x,t)\Psi^+(x,0)\rangle \sim \quad t^{-1} \quad \Longleftrightarrow \quad t^{-(1+\gamma^2)}$

density of states:



Out of equilibrium: sudden interaction quench (SQ) by M. A. Cazalilla: $\gamma_{eq} \neq \gamma_{SQ}$

Time dependent interaction quench of a Luttinger liquid

Effective Hamiltonian for a LL after bosonization:

$$H = \sum_{q \neq 0} \omega(q) b_q^\dagger b_q + \frac{g(q,t)}{2} [b_q b_{-q} + b_q^+ b_{-q}^+]$$

$\omega(q) = v|q|$, interaction $g_2(q) = g_2(q)|q|Q(t)$ couples left and right movers, $Q(t)$ the quench protocol, e.g. $Q(t) = t\Theta(t(\tau - t))/\tau + \Theta(t-\tau)$ for a linear quench.

Heisenberg equation of motion: q mode is only coupled to $-q$

$\Longrightarrow b_q(t) = u(q,t)\ b_q(0) + v^*(q,t)\ b_{-q}^+(0)$

non-Hermitian Landau-Zener problem: $i\partial_t \begin{bmatrix} u(q,t) \\ v(q,t) \end{bmatrix} = \begin{bmatrix} \omega(q) & g(q,t) \\ -g(q,t) & -\omega(q) \end{bmatrix} \begin{bmatrix} u(q,t) \\ v(q,t) \end{bmatrix}$

with $u(q,0) = 1$, $v(q,0) = 0$, $|u(q,t)|^2 - |v(q,t)|^2 = 1$.

General solution is difficult, let's proceed perturbatively!

$u(q,t) \approx \exp(-i\omega(q)t)$ and $v(q,t>0) \approx i \int_0^t dt' g(q,t') \exp(i\omega(q)(t-2t'))$

Heating

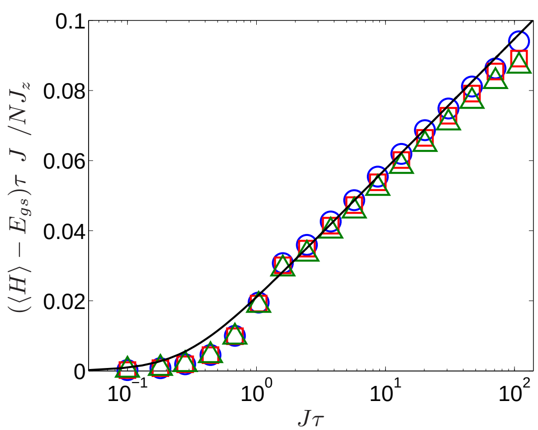
After a linear quench ($t > \tau$): $\langle H \rangle = \sum_{q \neq 0} \omega(q) \frac{g_2(q)^2}{2v^2} \left[\left(\frac{\sin(\omega(q)\tau)}{\omega(q)\tau} \right)^2 - 1 \right]$

$$\langle H \rangle = E_{gs} \left[1 - \left(\frac{\tau_0}{\tau} \right)^2 \ln \left(1 + \left(\frac{\tau}{\tau_0} \right)^2 \right) \right],$$

$E_{gs} = -L g_2^2 / 4\pi v R_0^2$ the adiabatic g.s. energy, $g_2(q) = g_2 \exp(-R_0|q|/2)$, $\tau_0 \equiv R_0/2v$.

For near adiabatic, linear quenches ($\tau \to \infty$), $\langle H \rangle - E_{gs} \sim \dfrac{\ln(\tau/\tau_0)}{\tau^2}$.

Numerical analysis of the XXZ Heisenberg model with a J_z quench using iTEBD.



The spatial and temporal behaviour of the one-particle density matrix

$G_r(x,t) \equiv \langle \Psi_r^+(x,t)\Psi_r(0,t)\rangle$

$G_r^0(x) = \dfrac{i}{2\pi(x+i\alpha)}$ is the free propagator.

For $t \gg \tau$ (steady state): $$\frac{G_r(x,t)}{G_r^0(x)} \sim \begin{cases} A\left(\tau/\tau_0\right)\left(\dfrac{R_0}{\min\{|x|, 2vt\}}\right)^{\gamma_{\rm SQ}} & \text{for } |x| \gg 2v\tau, \\ \left(\dfrac{R_0}{|x|}\right)^{\gamma_{\rm ad}} & \text{for } |x| \ll 2v\tau, \end{cases},$$

- $\gamma_{\rm SQ} = g_2^2/v^2 + \ldots$ and $\gamma_{\rm ad} = g_2^2/2v^2 + \ldots$ are the perturbative sudden quench and adiabatic exponents.

- Adiabatic enhancement factor $A\left(\tau/\tau_0\right)$: for SQ, $A(\tau \ll \tau_0) \sim 1$, while for near adiabatic quench, $A(\tau > \tau_0) \sim (\tau/\tau_0)^{\gamma_{ad}}$.

Momentum distribution

At $T = 0$ and finite $t \gg \tau$: $n(k,t)$ exhibits a jump of size $\sim Z(t) \sim$ at $k = k_F$, while it approximately scales for $|\tilde{k}| \gg 1/2vt$ as

$$n(k) - \frac{1}{2} \sim -\text{sign}(\tilde{k}) \times \begin{cases} A(\tau/\tau_0)\,|\tilde{k}R_0|^{\gamma_{\text{SQ}}}, & |\tilde{k}| \ll \dfrac{1}{2v\tau}\,, \\ |\tilde{k}R_0|^{\gamma_{\text{ad}}}, & |\tilde{k}| \gg \dfrac{1}{2v\tau}, \end{cases},$$

$\tilde{k} \equiv k - k_F$, $|\tilde{k}| \ll k_F$, $A(\tau/\tau_0) \sim \max\left[1, (\tau/\tau_0)^{\gamma_{ad}}\right]$, $\gamma_{\text{SQ}} = g_2^2/v^2 + \dots$ and $\gamma_{\text{ad}} = g_2^2/2v^2 + \dots$.

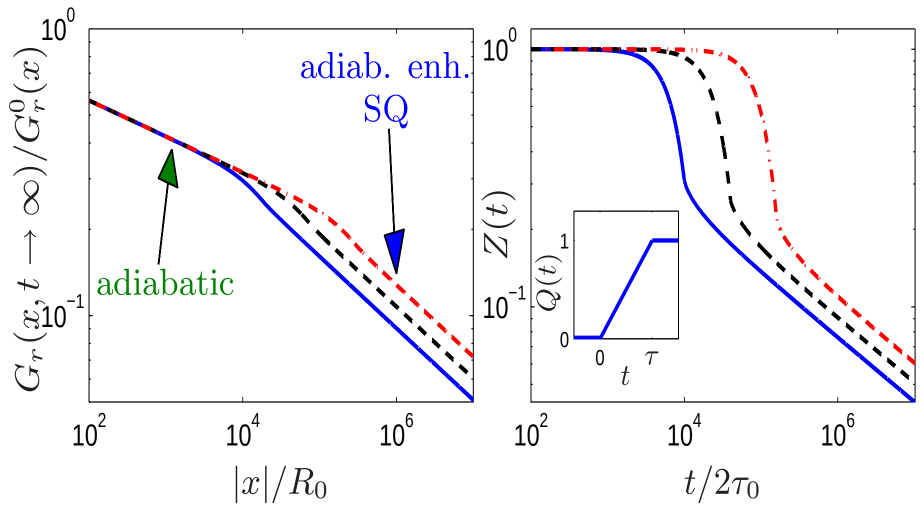


Figure 1: Left: the long time ($\tau \ll t \to \infty$), steady state limit of the one-particle density matrix is plotted on loglog scale for a linear quench and $2g_2 = v$ as a function of $|x|$, exhibiting the crossover from adiabatic behavior at small $|x|$ to SQ behavior with adiabatic enhancement at large $|x|$. The curves are plotted for $\tau/2\tau_0 = 10^4$, 4×10^4 and 16×10^4 from bottom to top in both panels. Right: Landau's quasiparticle weight, $Z(t)$ is plotted on a loglog scale as a function of t, bridging between the weakly interacting Fermi liquid to strongly suppressed $Z \ll 1$ with adiabatic enhancement. Inset: the linear quench protocol is shown.

Spatial-temporal "phase diagram"

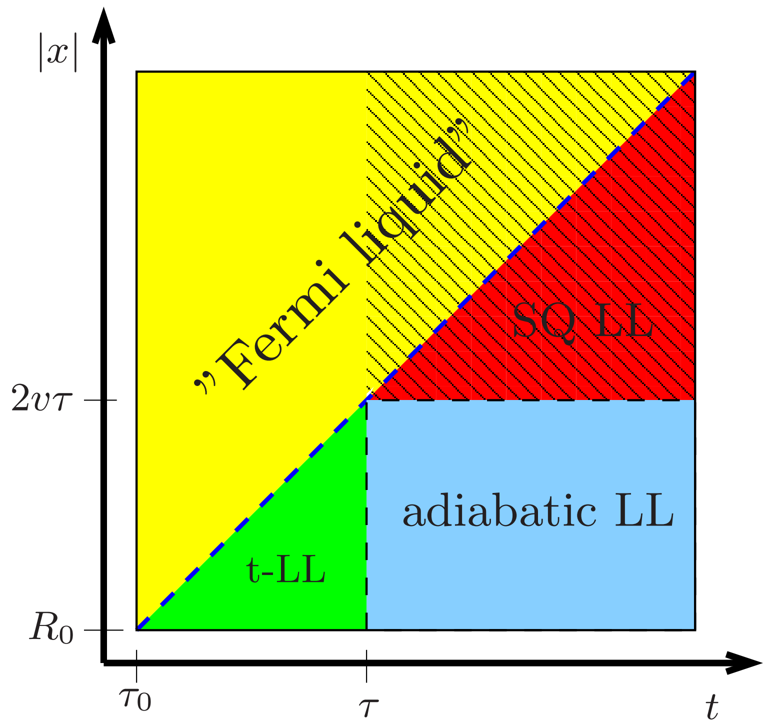


Figure 2: The schematic universal spatial-temporal characteristics of a quenched LL, with the boundaries denoting *crossovers*. In the adiabatic LL regime, the LL exponent of the final state, γ_{ad} governs spatial correlations, while in the SQ LL region, correlations decay with the SQ exponent, $\gamma_{\text{SQ}} > \gamma_{\text{ad}}$. Correlations are adiabatically increased by an amplitude, $A \sim (\tau/\tau_0)^{\gamma_{\text{ad}}}$ in the shaded region. In the Fermi liquid region a time dependent quasiparticle residue is found, while in the time-dependent LL (t-LL) region a quench protocol-dependent weakly interacting LL is found with a time dependent exponent. The dashed line denotes $|x| = 2vt$, i.e. the light-cone. For $\tau \ll \tau_0$, the SQ physics dominates everywhere.

Statistics of work, GGE

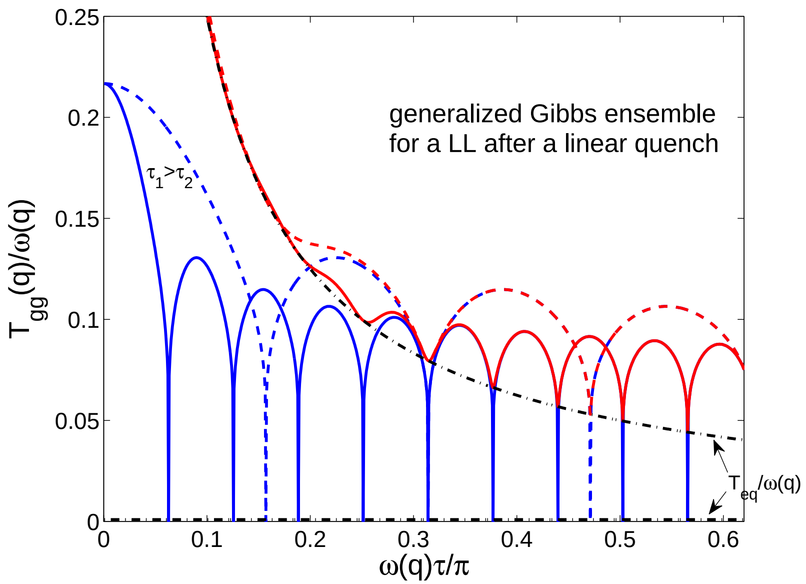
Characteristic function of work done: $G(\lambda,\tau) = \langle \exp[i\lambda H_H(t>\tau)]\exp[-i\lambda H_H(0)]\rangle$.

For a LL: $G(\lambda,\tau) = \exp\left[i\lambda E_{ad} - \sum_{q>0} \ln\left(1 + n_q(1-e^{2i\,\Omega_q\lambda})\right)\right]$,

where $E_{ad} = E_f - E_i$, n_q the occupation number in the final state, $\Omega_q = \sqrt{\omega_q^2(t>\tau) - g_q^2(t>\tau)}$.

The density matrix of the steady state ($t\to\infty$) à la Gibbs (use the integrals of motion with Lagrange multipliers):

$$\hat\rho_G = \frac{1}{Z_G}\prod_{q>0}\exp\left[-\beta_q\Omega_q\hat n_q\right]\delta_{\hat n_q,\hat n_{-q}}$$



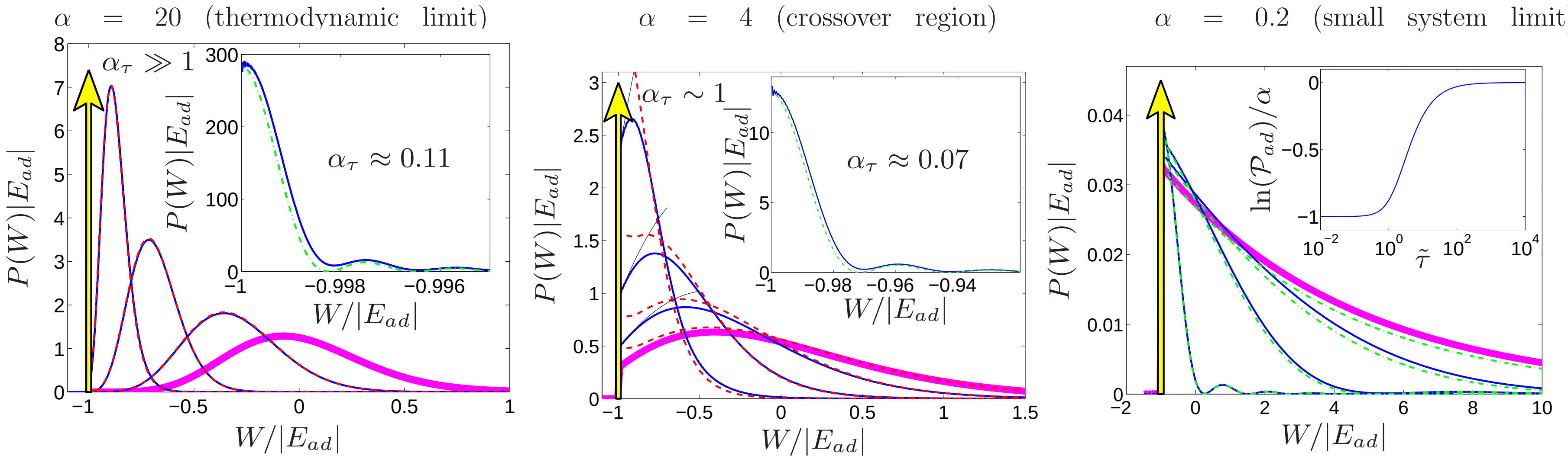


Figure 3: (Color online) The PDF of work done on a LL is plotted after a linear quench. Left panel: $\alpha = 20$ with $\tilde{\tau} = 0$, 1, 2.5 and 5 from right to left and 180 (inset, $P(W > E_{ad})$ only); middle panel: $\alpha = 4$ with $\tilde{\tau} = 0$, 1, 2 and 4 with increasing peak height and 55 (inset); right panel: $\alpha = 0.2$ with $\tilde{\tau} = 0$, 2, 5, and 25 from right to left. The thick magenta line denotes the exact SQ expression. The vertical arrow at $W = E_{ad}$ denotes the Dirac-delta peak, whose spectral weight $\mathcal{P}_{ad}$ is shown in the inset of the right panel on semilog scale as a function of the ramp time τ.

Many-body orthogonality exponents

Conclusion

- Solution of the Luttinger model for a general interaction quench.
- Several distinct spatial-temporal regions revealed by correlation functions.
- Good agreement with numerics: Luttinger model probably applicable out of equilibrium.
- With minor modifications, applicable to 1D bosons, spins etc.
- Ultracold fermionic gases (^{40}K, ^{6}Li, ^{171}Yb-^{173}Yb, ^{163}Dy, ^{87}Sr) at low temperatures ($T < 0.1\ E_F$) available.
- Loschmidt echo, fidelity?